higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

MARKING GUIDELINE

NATIONAL CERTIFICATE

MATHEMATICS N6

29 MARCH 2019

This marking guideline consists of 16 pages.

The paper is marked out of 200 and divided by 2 to get a mark out of 100.

QUESTION 1

1.1 $z = \tan(xy)$

$$\frac{\partial z}{\partial x} = \sec^2(xy).y \quad \checkmark\checkmark$$

$$\frac{\partial^2 z}{\partial x^2} = y.2\sec(xy).\sec(xy)\tan(xy).y \quad \checkmark\checkmark\checkmark\checkmark$$

(6)

1.2

$$r = \sqrt{x^2 + y^2} \quad \checkmark$$

$$\Delta r = \frac{\partial r}{\partial x}\Delta x + \frac{\partial r}{\partial y}\Delta y \quad \checkmark$$

$$= \frac{x}{\sqrt{x^2 + y^2}}\Delta x + \frac{y}{\sqrt{x^2 + y^2}}\Delta y \quad \checkmark\checkmark$$

$$= \frac{20}{\sqrt{20^2 + 10^2}}.2 + \frac{10}{\sqrt{20^2 + 10^2}}(-2) \quad \checkmark$$

$$= \frac{40 - 20}{\sqrt{20^2 + 10^2}} \quad \checkmark$$

$$= 0,894\,m$$

(6)

[12]

QUESTION 2

2.1

$$\int y dx$$
$$= \int 1 - \tan^4 3x dx$$
$$= \int (1 - \tan^2 3x)\left(1 + \tan^2 3x\right) dx \quad ✓✓$$
$$= \int (1 - \tan^2 3x)\left(\sec^2 3x\right) dx \quad ✓$$
$$= \int (\sec^2 3x - \tan^2 3x \sec^2 3x dx \quad ✓✓$$
$$= \frac{1}{3}\tan 3x - \frac{1}{3}\frac{\tan^3 3x}{3} + c \quad ✓✓✓$$

Or using $u = \tan 3x$

Alternative 1

$$\int y dx$$
$$= \int 1 - \tan^4 3x dx$$
$$\int (1 - \tan^2 3x . \tan^2 3x dx \quad ✓$$
$$= \int (1 - \tan^2 3x\left(\sec^2 3x - 1\right) dx \quad ✓$$
$$= \; = \int (1 - \tan^2 3x \sec^2 3x + \tan^2 3x dx \quad ✓✓$$
$$= x - \frac{1}{3}\frac{\tan^3 3x}{3} + \frac{1}{3}\tan 3x - x + c \quad ✓✓✓$$
$$= \frac{1}{3}\frac{\tan^3 3x}{3} + \frac{1}{3}\tan 3x + c \quad ✓$$

Alternative 2

$$\int 1 - \frac{\sin^4 3x}{\cos^4 3x} dx \quad ✓$$
$$\int \frac{\cos^4 3x - \sin^4 3x}{\cos^4 3x} dx \quad ✓$$
$$= \int \frac{(\cos^2 3x + \sin^2 3x)(\cos^2 3x - \sin^2 3x)}{\cos^4 3x} dx \quad ✓$$
$$= \int \frac{\cos^2 3x - \sin^2 3x}{\cos^4 3x} dx \quad ✓$$
$$= \int \frac{\cos^2 3x}{\cos^4 3x} - \frac{\sin^2 3x}{\cos^4 3x} dx \quad ✓$$
$$= \int \sec^2 3x - \tan^2 3x \sec^2 3x dx \quad ✓$$
$$= \frac{1}{3}\tan 3x - \frac{1}{9}\tan^3 3x + c \quad ✓✓$$

(8)

2.2 $\int x(\ln x)^2\,dx$

$= \frac{x^2}{2}(\ln x)^2 - \int 2\ln x . \frac{1}{x} . \frac{x^2}{2}\,dx$ ✓✓✓

$= \frac{x^2}{2}(\ln x)^2 - \int x \ln x\,dx$ ✓

$= \frac{x^2}{2}(\ln x)^2 - \left[\frac{x^2}{2}\ln x - \int \frac{1}{x} . \frac{x^2}{2}\,dx\right]$ ✓✓

$= \frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \frac{1}{2}\int x\,dx$ ✓

$= \frac{x^2}{2}(\ln x)^2 - \frac{x^2}{2}\ln x + \frac{1}{2} . \frac{x^2}{2} + c$ ✓

(8)

2.3 $\int (1-2\sin^2 2x)^3\,dx$

$= \int (\cos 4x)^3\,dx$ ✓✓

$= \int \cos^2 4x \cos 4x\,dx$ ✓

$= \int (1-\sin^2 4x)\cos 4x\,dx$ ✓

$= \int (\cos 4x - \sin^2 4x \cos 4x\,dx$ ✓

$= \frac{\sin 4x}{4} - \frac{1}{4}\frac{\sin^3 4x}{3} + c$ ✓✓✓

> $\int \left[1-2(1-\cos^2 2x)\right]^3 dx$
>
> $= \int \left[-1+2\cos^2 2x)\right]^3 dx$
>
> $= \int (\cos 4x)^3\,dx$

> Or using $u = \sin 4x$

(8)

2.4

$$\int \frac{\sin 2x}{e^{2x}} dx$$

$$\boxed{f(x) = e^{-2x} \quad g'(x) = \sin 2x}$$

$$= \int e^{-2x} \sin 2x dx \quad ✓$$

$$= e^{-2x} . -\frac{\cos 2x}{2} - \int -2e^{-2x} . -\frac{\cos 2x}{2} dx \quad ✓✓$$

$$= -\frac{1}{2} e^{-2x} \cos 2x - \int e^{-2x} \cos 2x dx$$

$$= -\frac{1}{2} e^{-2x} \cos 2x - \left[e^{-2x} . \frac{\sin 2x}{2} - \int -2e^{-2x} . \frac{\sin 2x}{2} dx \right] \quad ✓✓$$

$$= -\frac{1}{2} e^{-2x} \cos 2x - \frac{1}{2} e^{-2x} . \sin 2x - \int e^{-2x} \sin 2x dx$$

$$2\int e^{-2x} \sin 2x dx = -\frac{1}{2} e^{-2x} \cos 2x - \frac{1}{2} e^{-2x} . \sin 2x \quad ✓$$

$$\int e^{-2x} \sin 2x dx = -\frac{1}{4} e^{-2x} \cos 2x - \frac{1}{4} e^{-2x} . \sin 2x + c \quad ✓✓$$

Alternative

$$\boxed{f(x) = \sin 2x \qquad g'(x) = e^{-2x}}$$

$$\int e^{-2x} \sin 2x dx \quad ✓$$

$$= \sin 2x \frac{e^{-2x}}{-2} - \int 2 \cos 2x \frac{e^{-2x}}{-2} dx \quad ✓✓$$

$$= -\frac{1}{2} \sin 2x \, e^{-2x} + \int e^{-2x} \cos 2x \, dx$$

$$= -\frac{1}{2} \sin 2x \, e^{-2x} + \left[\cos 2x \frac{e^{-2x}}{-2} - \int -2 \sin 2x \frac{e^{-2x}}{-2} dx \right] \quad ✓✓$$

$$= -\frac{1}{2} \sin 2x \, e^{-2x} - \frac{1}{2} e^{-2x} \cos 2x - \int e^{-2x} \sin 2x \, dx$$

$$2\int e^{-2x} \sin 2x \, dx = -\frac{1}{2} \sin 2x \, e^{-2x} - \frac{1}{2} e^{-2x} \cos 2x + c \quad ✓$$

$$\int e^{-2x} \sin 2x \, dx = -\frac{1}{4} \sin 2x \, e^{-2x} - \frac{1}{4} e^{-2x} \cos 2x + c \quad ✓✓$$

(8)

2.5

$$\int \frac{x^2-2}{x^4-4}dx$$

$$= \int \frac{x^2-2}{(x^2+2)(x^2-2)}dx \quad \checkmark$$

$$= \int \frac{1}{2+x^2}dx \quad \checkmark$$

$$= \frac{1}{\sqrt{2}}\tan^{-1}\frac{x}{\sqrt{2}}+c \quad \checkmark\checkmark$$

(4)

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QUESTION 3

3.1

$$\int \frac{x^2+x-7}{x^2+x-6}dx \quad \checkmark$$

$$\int \frac{(x^2+x-6)-1}{x^2+x-6}dx \quad \checkmark\checkmark$$

$$\int 1-\frac{1}{x^2+x-6}dx \quad \checkmark$$

Or using long division

$$\frac{1}{x^2+x-6}=\frac{1}{(x+3)(x-2)}=\frac{A}{(x+3)}+\frac{B}{(x-2)}$$

$$1=A(x-2)+B(x+3) \quad \checkmark$$

$$x=2 \quad B=\frac{1}{5} \quad \checkmark$$

$$x=-3 \quad A=-\frac{1}{5} \quad \checkmark$$

$$1=A(x-2)+B(x+3)$$
$$1=Ax-2A+Bx+3B$$
$$A+B=0\text{..........}(1) \Rightarrow B=-A$$
$$-2A+3B=1\text{.........}(2)$$
$$-2A+3(-A)=1$$
$$-5A=1 \Rightarrow A=-\frac{1}{5} \quad and \quad B=\frac{1}{5}$$

$$\int 1-\frac{1}{x^2+x-6}dx=\int 1-\left[\frac{-\frac{1}{5}}{(x+3)}+\frac{\frac{1}{5}}{(x-2)}\right]dx \quad \checkmark\checkmark$$

$$= x+\frac{1}{5}\ln \ln(x+3)-\frac{1}{5}\ln(x-2)+c \quad \checkmark\checkmark\checkmark$$

$$= x+\frac{1}{5}\{(\ln(x+3)-\ln(x-2)\}+c$$

$$= x+\frac{1}{5}\ln\frac{x+3}{x-2}+c$$

(12)

3.2 $\int \frac{7x^2-12x+8}{(2x-1)(x^2-2x+2)}dx$

$\frac{7x^2-12x+8}{(2x-1)(x^2-2x+2)} = \frac{A}{2x-1} + \frac{Bx+C}{x^2-2x+2}$ ✓✓

$7x^2-12x+8 = A(x^2-2x+2)+(Bx+C)(2x-1)$ ✓

$x=\frac{1}{2} \quad 7(\frac{1}{2})^2-12(\frac{1}{2})+8 = A\{(\frac{1}{2})^2-2(\frac{1}{2})+2\} \Rightarrow$ ✓ $\boxed{A=3}$ ✓

$7x^2-12x+8 = Ax^2-2Ax+2A+2Bx^2+2Cx-Bx-C$ ✓

$A+2B=7 \qquad 3+2B=7$ $\boxed{B=2}$ ✓

$-2A+2C-B=-12$

$-2(3)+2C-2=-12$ $\boxed{C=-2}$ ✓

$\int \frac{7x^2-12x+8}{(2x-1)(x^2-2x+2)}dx$

$=\int \frac{3}{2x-1}+\frac{2x-2}{x^2-2x+2}dx$ ✓

$=\frac{3}{2}\ln(2x-1)+\ln(x^2-2x+2)+c$ ✓✓✓

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QUESTION 4

4.1 $\frac{1}{x}\frac{dy}{dx}-\frac{1}{x^2}y = 3x-\sin x+\cos x$

$\frac{dy}{dx}-\frac{1}{x}y = x(3x-\sin x+\cos x)$ ✓

$e^{\int pdx} = e^{\int -\frac{1}{x}dx}$ ✓

$= e^{-\ln x}$ ✓

$= e^{\ln x^{-1}}$

$= x^{-1} \quad = \frac{1}{x}$ ✓

$$\boxed{e^{-\ln x} = \frac{1}{e^{\ln x}} = \frac{1}{x}}$$

$$\int Qe^{\int pdx}dx = \int x(3x - \sin x + \cos x)\frac{1}{x}dx \quad ✓$$

$$= \int 3x - \sin x + \cos x)dx \quad ✓$$

$$= \frac{3}{2}x^2 + \cos x + \sin x \quad ✓✓$$

$$\frac{y}{x} = \frac{3}{2}x^2 + \cos x + \sin x + c \quad ✓$$

$$x = 1;\ y = 1 \quad \frac{1}{1} = \frac{3}{2}(1)^2 + \cos(1) + \sin(1) + c \quad ✓$$

$$c = -1,882 \quad ✓$$

$$\frac{3}{2}x^2 + \cos x + \sin x - 1,882 \quad ✓$$

$$\frac{y}{x} = \frac{3}{2}x^2 + cosx + sinx - 1.882$$

(12)

4.2

$$6\frac{d^2y}{dx^2} - \frac{dy}{dx} - y = x^2$$

$$6r^2 - r - 1 = 0$$

$$(3r+1)(2r-1) = 0 \quad ✓$$

$$r = -\frac{1}{3} \quad r = \frac{1}{2} \quad ✓$$

$$y_c = Ae^{-\frac{1}{3}x} + Be^{\frac{1}{2}x} \quad ✓$$

$$y = Cx^2 + Dx + E \quad ✓$$

$$\frac{dy}{dx} = 2Cx + D \qquad \frac{d^2y}{dx^2} = 2C \quad ✓$$

$$6(2C) - (2Cx + D) - (Cx^2 + Dx + E) = x^2 \quad ✓$$

$$12C - 2Cx - 2D - Cx^2 - Dx - E = x^2$$

$$x^2 \ :\ -C = 1 \quad \therefore C = -1 \quad ✓$$

$$x:\ -2C - 2D = 0 \quad \therefore D = 2 \quad ✓$$

$$12C - D - E = 0 \quad \therefore E = -14 \quad ✓$$

$$y_p = -x^2 + 2x - 14 \quad ✓$$

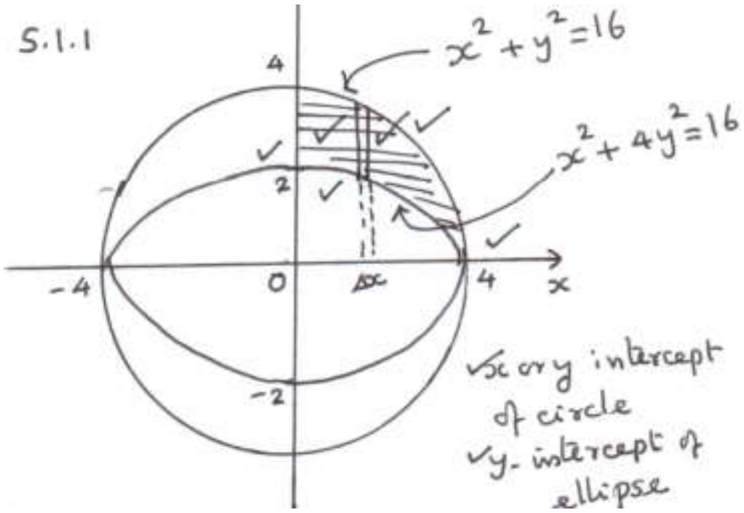
$$y = y_c + y_p$$

$$y = Ae^{-\frac{1}{3}x} + Be^{\frac{1}{2}x} - x^2 + 2x - 14 \quad ✓$$

(12)
[24]

QUESTION 5

5.1 5.1.1



(6)

5.1.2 $Area =$

$$= \int_a^b y_1 - y_2 \, dx \quad ✓$$

$$= \int_0^4 \sqrt{16-x^2} - \frac{1}{2}\sqrt{16-x^2} \, dx \quad ✓✓✓$$

$$= \frac{1}{2}\int_0^4 \sqrt{16-x^2} dx \quad ✓$$

$$= \frac{1}{2}\left[\frac{16}{2}\sin^{-1}\frac{x}{4} + \frac{x}{2}\sqrt{16-x^2}\right]_0^4 \quad ✓✓$$

$$= \frac{1}{2}\left[8\sin^{-1}\frac{4}{4} + \frac{4}{2}\sqrt{16-4^2} - \left\{8\sin^{-1}\frac{0}{4} + \frac{0}{2}\sqrt{16-0^2}\right\}\right] \quad ✓✓$$

$$= \frac{1}{2}\left[8\sin^{-1}1\right] \quad = 6,283 \ \ or \ \ 2\pi \ \ units^2 \quad ✓$$

,

(10)

5.1.3

$$\bar{x} = \frac{A_{m-y}}{A}$$

$$A_{m-y} = \int_a^b r\,dA$$

$$= \int_0^4 x\left[\sqrt{16-x^2} - \frac{1}{2}\sqrt{16-x^2}\right]dx \quad ✓✓✓$$

$$= \frac{1}{2}\int_0^4 x\sqrt{16-x^2}\,dx \quad ✓$$

$$= \frac{1}{2}(-\frac{1}{2})\int_0^4 -2x(16-x^2)^{\frac{1}{2}}\,dx \quad ✓$$

$$= -\frac{1}{4}\left[\frac{\left(16-x^2\right)^{\frac{3}{2}}}{\frac{3}{2}}\right]_0^4 \quad ✓$$

$$= -\frac{1}{6}\left[\left(16-x^2\right)\right]_0^4$$

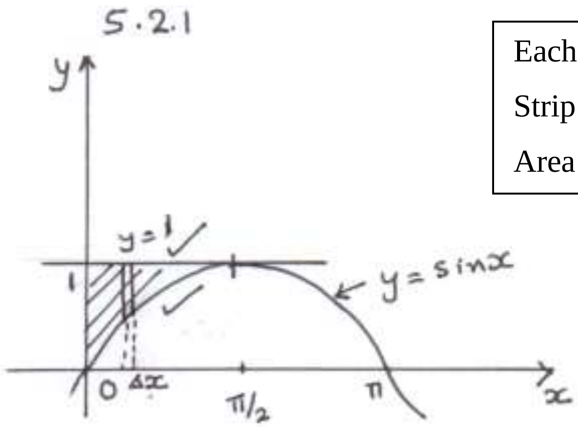
$$= -\frac{1}{6}\left[\left(16-4^2\right)^{\frac{3}{2}} - (16-0^2)^{\frac{3}{2}}\right] \quad ✓$$

$$= \frac{32}{3}units^2 \quad or \quad 10,667 units^2 \quad ✓$$

$$\bar{x} = \frac{32/3}{6,283} = 1,698 \text{ units} \quad ✓✓$$

(10)

5.2 5.2.1



Each grapl ✓✓
Strip ✓
Area ✓

(4)

5.2.2

$$V_x = \pi \int_a^b y_1^2 - y_2^2 \quad ✓$$

$$= \pi \int_0^{\frac{\pi}{2}} 1 - \sin^2 x dx \quad ✓✓ \qquad or \quad \pi \left[x - \left(\frac{x}{2} - \frac{\sin 2x}{4} \right) \right]_0^{\frac{\pi}{2}}$$

$$= \pi \int_0^{\frac{\pi}{2}} \cos^2 x dx \quad ✓ \qquad = \pi \left[x - \frac{x}{2} + \frac{\sin 2x}{4} \right]_0^{\frac{\pi}{2}}$$

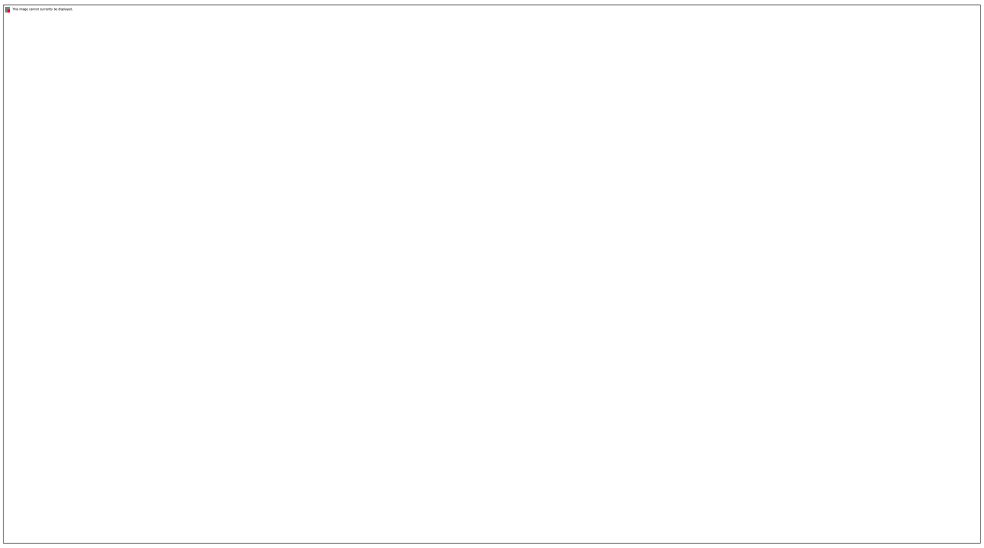
$$= \pi \left[\frac{x}{2} + \frac{\sin 2x}{4} \right]_0^{\frac{\pi}{2}} \quad ✓ \qquad = \pi \left[\frac{x}{2} + \frac{\sin 2x}{4} \right]_0^{\frac{\pi}{2}}$$

$$= \pi \left[\frac{\pi}{4} + \frac{\sin 0}{4} - \{0\} \right] \quad ✓✓$$

$$= \frac{\pi^2}{4} \text{units}^3 \ \ (\text{or } 2{,}467) \quad ✓$$

(8)

5.3 5.3.1



(6)

5.3.2

$$V_x = \pi \int_a^b y_1^2 - y_2^2 \quad ✓$$

$$= \pi \int_{-1}^{1} (e^x)^2 dx \quad ✓✓$$

$$= \pi \left[\frac{e^{2x}}{2} \right]_{-1}^{1} \quad ✓$$

$$=3{,}627\pi \quad \text{or} \quad 11{,}394\text{units}^3$$

$$= \pi \left[\frac{e^2}{2} - \frac{e^{-2}}{2} \right] \quad ✓ \qquad or \ = \frac{\pi}{2}\left[e^2 \text{-} e^{-2} \right]$$

$$=3{,}627\pi \quad \text{or} \quad 11{,}394\text{units}^3 \quad ✓$$

(6)

5.3.3

$$I_x = \frac{1}{2}\pi\rho\int_a^b y^4 dx \quad \checkmark\checkmark$$

$$= \frac{1}{2}\pi\rho\int_{-1}^{1} (e^x)^4 dx \quad \checkmark\checkmark$$

$$= \frac{1}{2}\pi\rho\left[\frac{e^{4x}}{4}\right]_{-1}^{1} \quad \checkmark$$

$$= \frac{1}{2}\pi\rho\left[\frac{e^4}{4} - \frac{e^{-4}}{4}\right] \quad or\ \frac{1}{8}\pi\rho\left(e^4 - e^{-4}\right) \quad \checkmark$$

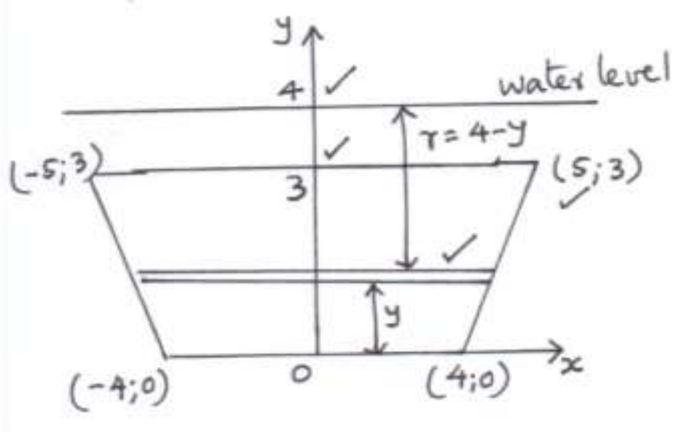
$$= 6{,}823\pi\rho \quad or\ 21{,}434\rho \quad \checkmark$$

$$= \frac{6{,}823\pi m}{3{,}627\pi} = \frac{21{,}434m}{11{,}394} \quad \checkmark\checkmark$$

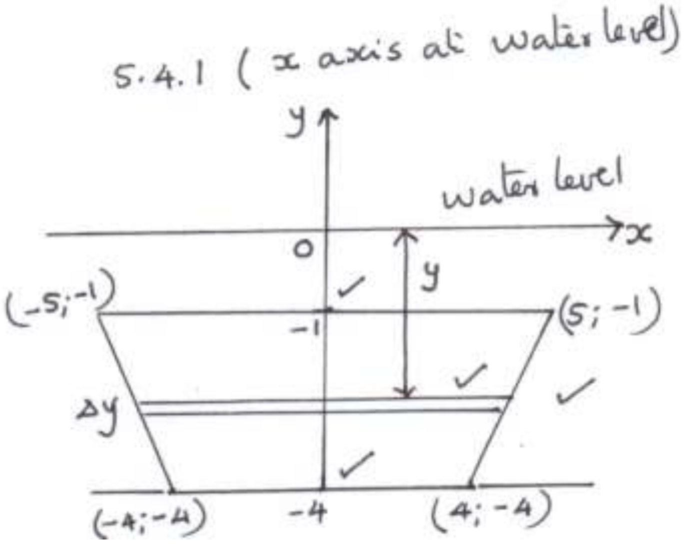
$$= 1{,}881m \quad \checkmark$$

(10)

5.4 5.4.1



Alternative



(4)

5.4.2

$$m = \frac{3-0}{5-4} or \frac{0-3}{4-5} = 3 \quad ✓$$

$$y - 0 = 3(x-4) \quad or \quad y - 3 = 3(x-5)$$

$$y = 3x - 12 \quad ✓ \quad or \quad y - 3 = 3x - 15$$

$$or \quad y = 3x - 12$$

$$x = \frac{y+12}{3}$$

$$= \int_a^b r dA$$

$$= \int_0^3 (4-y)2\frac{y+12}{3} dy \quad ✓✓$$

$$= \frac{2}{3}\int_0^3 (4-y)(y+12) dy$$

$$= \frac{2}{3}\int_0^3 (-8y - y^2 + 48) dy \quad ✓$$

$$= \frac{2}{3}\left[-4y^2 - \frac{y^3}{3} + 48y\right]_0^3 \quad ✓$$

$$= \frac{2}{3}\left[-4(3)^2 - \frac{(3)^3}{3} + 48(3) - \{0\}\right] \quad ✓$$

$$= 66m^3 \quad ✓$$

Alternative

With x-axis at the water level

$$m = \frac{-1+4}{5-4} or \frac{-4+1}{4-5} = 3 \quad ✓$$

$$y + 4 = 3(x-4) \quad or \quad y + 1 = 3(x-5)$$

$$y = 3x - 16 \quad ✓ \quad or \quad y = 3x - 15 - 1$$

$$or \quad y = 3x - 16$$

$$x = \frac{y+16}{3}$$

First moment of area $= \int_a^b r\,dA$

$$= \int_{-4}^{-1} y2\frac{y+16}{3}dy \quad ✓✓$$

$$= \frac{2}{3}\int_{-4}^{-1} y^2 + 16y\,dy \quad ✓$$

$$= \frac{2}{3}\left[\frac{y^3}{3} + 8y^2\right]_{-4}^{-1} \quad ✓$$

$$= \frac{2}{3}\left[\frac{-1}{3} + 8 - \{\frac{-64}{3} + 8.16\}\right] \quad ✓$$

$$= -66m^3 \quad ✓$$

(8)

5.4.3 Second moment of area

$$= \int_a^b r^2 dA$$

$$= \int_0^3 (4-y)^2 2\frac{y+12}{3}dy \quad ✓✓$$

$$= \frac{2}{3}\int_0^3 (4-y)^2 (y+12)dy$$

$$= \frac{2}{3}\int_0^3 (16-8y+y^2)(y+12)dy$$

$$= \frac{2}{3}\int_0^3 (-80y+4y^2+y^3+192)dy \quad ✓$$

$$= \frac{2}{3}\left[-40y^2 + \frac{4}{3}y^3 + \frac{y^4}{4} + 192y\right]_0^3 \quad ✓$$

$$= \frac{2}{3}\left[-40(3)^2 + \frac{4}{3}(3)^3 + \frac{(3)^4}{4} + 192(3) - \{0\}\right] \quad ✓$$

$$= 181,5m^4 \quad ✓$$

$$\boxed{\bar{\bar{y}} = \frac{181,5}{66} ✓ = 2,75m ✓}$$

Alternative

Second moment of area

$$= \int_a^b r^2 dA$$

$$= \int_{-4}^{-1} y^2 \, 2\frac{y+16}{3} dy \quad ✓✓$$

$$= \frac{2}{3}\int_{-4}^{-1} y^3 + 16y^2 dy \quad ✓$$

$$= \frac{2}{3}\left[\frac{y^4}{4} + \frac{16y^3}{3}\right]_{-4}^{-1} \quad ✓$$

$$= \frac{2}{3}\left[\frac{1}{4} - \frac{16}{3} - \left\{\frac{(-4)^4}{4} + \frac{16(-4)^3}{3}\right\}\right]_{-4}^{-1} \quad ✓$$

$$= 181,5m^4 \quad ✓$$

$$\overline{y} = \frac{181,5m^4}{-66m^3} = -2,75m \quad ✓✓$$

(8)

[80]

QUESTION 6

6.1 $x = e^{\theta}\sin\theta$

$$\frac{dx}{d\theta} = e^{\theta}\cos\theta + e^{\theta}\sin\theta \quad ✓$$

$$= e^{\theta}(\cos\theta + \sin\theta)$$

$$\left[\frac{dx}{d\theta}\right]^2 = e^{2\theta}(\cos\theta + \sin\theta)^2 \quad ✓$$

$$y = e^{\theta}\cos\theta \quad ✓$$

$$\frac{dy}{d\theta} = -e^{\theta}\sin\theta + e^{\theta}\cos\theta \quad ✓$$

$$= e^{\theta}(\cos\theta - \sin\theta)$$

$$\left[\frac{dy}{d\theta}\right]^2 = e^{2\theta}(\cos\theta - \sin\theta)^2 \quad ✓$$

$$\left[\frac{dx}{d\theta}\right]^2 + \left[\frac{dy}{d\theta}\right]^2 = e^{2\theta}\left[(\cos\theta + \sin\theta)^2 + (\cos\theta - \sin\theta)^2\right] \quad ✓$$

$$= e^{2\theta}\left[\cos^2\theta + \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta\right] \quad ✓$$

$$= e^{2\theta}[2] \quad ✓$$

Or using

$a^2+b^2=(a+b)^2-2ab \qquad e^{2\theta}\left[(\cos\theta+\sin\theta)^2+(\cos\theta-\sin\theta)^2\right] = e^{2\theta}[2]$

$$S=\int_{\theta_1}^{\theta_2}\sqrt{\left[\frac{dx}{d\theta}\right]^2+\left[\frac{dy}{d\theta}\right]^2}\,d\theta$$

$$=\int_0^{\frac{\pi}{3}}\sqrt{2e^{2\theta}}\,d\theta \quad ✓$$

$$=\sqrt{2}\int_0^{\frac{\pi}{3}}e^{\theta}\,d\theta$$

$$=\sqrt{2}\left[e^{\theta}\right]_0^{\frac{\pi}{3}} \quad ✓$$

$$=\sqrt{2}\left[e^{\frac{\pi}{3}}-e^0\right] \quad ✓$$

$$=2,616 \text{ units} \quad ✓$$

(12)

6.2

$$y=\frac{3}{2}x$$

$$\frac{dy}{dx}=\frac{3}{2} \quad ✓$$

$$\left[\frac{dy}{dx}\right]^2=\frac{9}{4} \quad ✓$$

$$1+\left[\frac{dy}{dx}\right]^2=1+\frac{9}{4}=\frac{13}{4} \quad ✓✓$$

$$A_x=2\pi\int_a^b y\sqrt{1+\left(\frac{dy}{dx}\right)^2}\,dx \quad ✓$$

$$=2\pi\int_2^4\frac{3}{2}x\sqrt{\frac{13}{4}}\,dx \quad ✓✓✓$$

$$=2\pi\,\frac{3}{2}\,\frac{\sqrt{13}}{2}\int_2^4 x\,dx \quad ✓$$

$$=\frac{3\sqrt{13}\pi}{2}\left[\frac{x^2}{2}\right]_2^4 \quad ✓$$

$$=\frac{3\sqrt{13}\pi}{4}\left[4^2-2^2\right] \quad ✓$$

$$=9\sqrt{13}\pi \text{ or } 101,945 \text{ units}^2 \quad ✓$$

$$x=\frac{2}{3}y$$

$$\frac{dx}{dy}=\frac{2}{3}$$

$$\left[\frac{dx}{dy}\right]^2=\frac{4}{9}$$

$$1+\left[\frac{dx}{dy}\right]^2=1+\frac{4}{9}=\frac{13}{9}$$

$$A_x=2\pi\int_a^b y\sqrt{1+\left(\frac{dx}{dy}\right)^2}\,dy$$

$$=2\pi\int_3^6 y\sqrt{\frac{13}{9}}\,dy$$

$$=2\pi\,\frac{\sqrt{13}}{3}\int_3^6 y\,dy$$

$$=\frac{2\sqrt{13}\pi}{3}\left[\frac{y^2}{2}\right]_3^6$$

$$=\frac{\sqrt{13}\pi}{3}\left[6^2-3^2\right]$$

$$=9\sqrt{13}\pi \text{ or } 101,945 \text{ units}^2$$

(12)
[24]

TOTAL: 200